

# Four-dimensional understanding of quantum mechanics.

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## Abstract

In this paper I will try to convince that quantum mechanics does not have to lead to indeterminism, but is just a natural consequence of four-dimensional nature of our world - that for example particles shouldn't be imagined as 'moving points' in space, but as their trajectories in the spacetime like in optimizing action formulation of Lagrangian mechanics. There will be analyzed simplified model - Boltzmann distribution among trajectories occurs to give quantum mechanic like behavior - for example electron moving in proton's potential would make some concrete trajectory which average exactly to the probability distribution of the quantum mechanical ground state. We will use this model to build intuition about quantum mechanics and discuss its generalizations to get some effective approximation of physics. We will see that topological excitations of the simplest model obtained this way already creates known from physics particle structure, their decay modes and electromagnetic/gravitational interactions between them.

## 1 Introduction

Quantum mechanics (QM) is probabilistic theory - it can predict only probability of events. While its stormy history there was developed understanding that this situation is inevitable - that e.g. electrons have to use some 'real random generator' to choose own behavior - that physics cannot be deterministic. The strongest argument are Bell's inequalities ([1]), which argue that QM probabilities cannot be achieved by some additional 'hidden variables' resulting measurement. The source of the problem are the squares in formulas for probability - in this paper I'll try to convince that they are just a natural result of four-dimensional nature of our spacetime.

Let's look at Lagrangian mechanics. It can be formulated by energy conserving evolution equations. There is also another formulation - by optimizing so called action: there is some functional assigning to each trajectory some value (its action) and e.g. particle going between some two points chooses trajectory optimizing this action. So intuitively

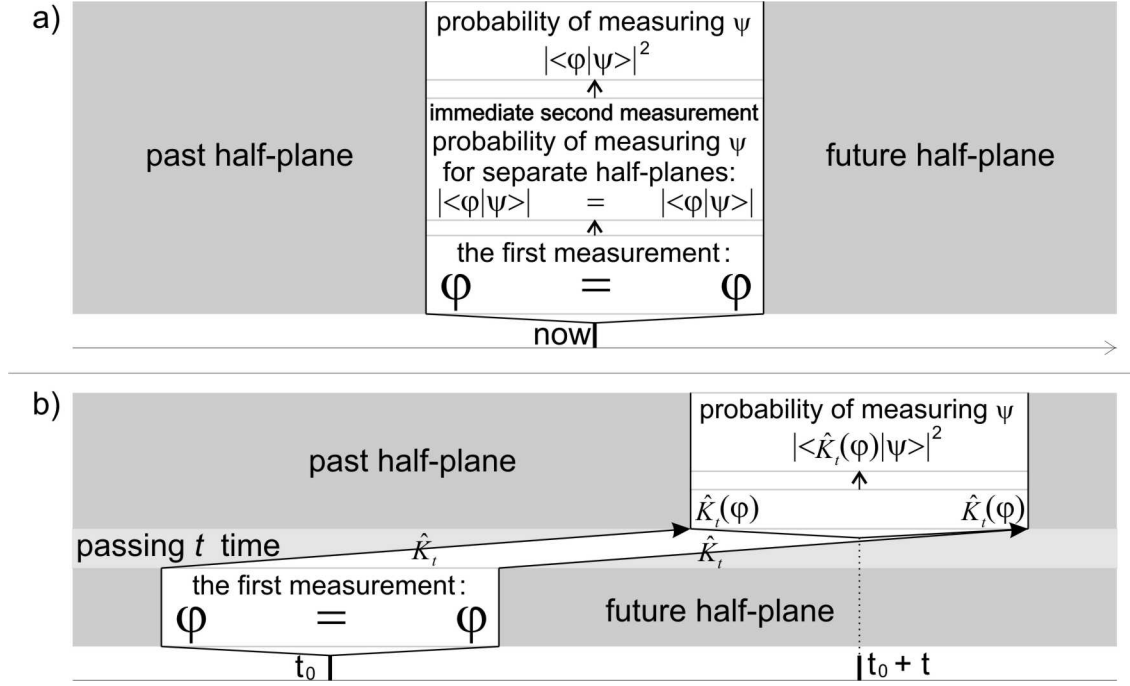


Figure 1: Four-dimensional understanding of quantum mechanics. a) measurement gives us some eigenvector (eigenfunction)  $\varphi$  of some hermitean operator. Now we make immediate second measurement in some different basis - for each separate half-planes the best estimation for probability of measuring  $\psi$  should be  $|\langle \varphi | \psi \rangle|$ . We also know that situation from both half-planes has to be the same in the moment, so finally we get the squares known from QM. On b) we wait  $t$  before making the second measurement: evolution of quantum state for both half-planes is given by some linear operator - the propagator.

in given moment **trajectory of particle minimizes stresses from both past and future time directions**, which leads through Euler-Lagrange equations to the evolution equations.

Let's take this intuition into QM. In given moment we make a measurement. We can divide spacetime into two half-planes: past and future one, joining in the given moment. The measurement fix some part of information, like direction of spin. Now if want to measure again after changing the basis, probability should be proportional to absolute value of corresponding coordinates - for both separate half-planes. We also know that both of them 'glues' in the moment - give the same value, so finally the probability while measurement should be proportional to the square of probability for separate half-planes as in quantum mechanics (fig. 1).

This intuitive explanation of the inconvenient squares seems to be in contradiction with our feeling that past and future are qualitatively completely different. From the other side there is common belief in CPT conservation which have to be fulfilled by Lorentz invariant local quantum field theories (CPT theorem [2]) as the Standard Model. This symmetry

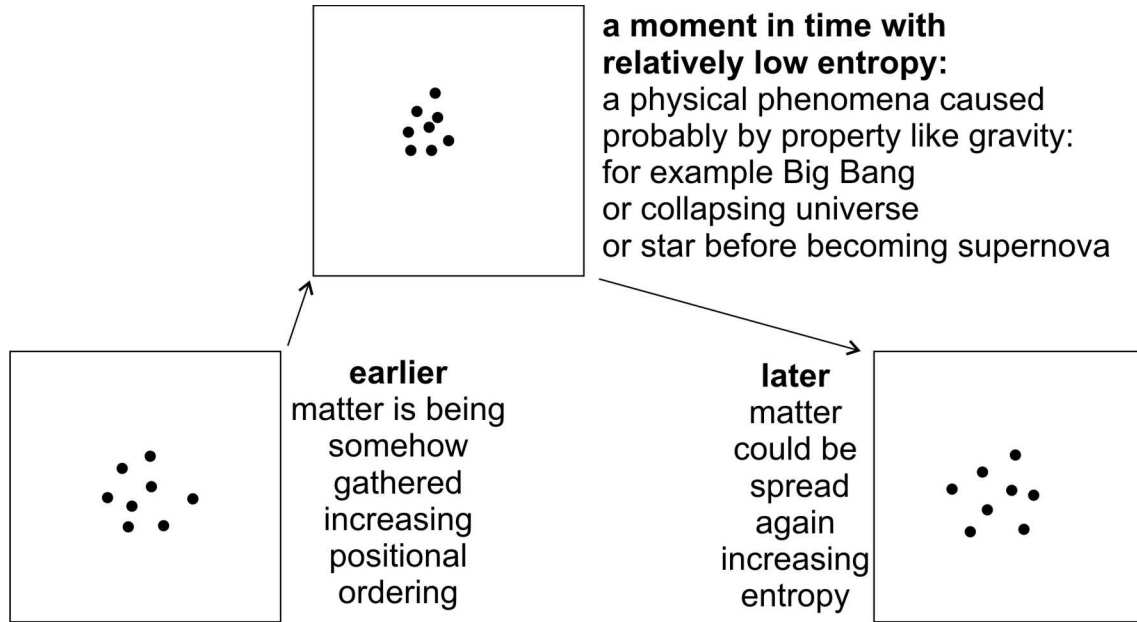


Figure 2: Second law of thermodynamics as naturally appearing property of models with e.g. always attracting gravity. This attraction has tendency to gather matter spread through some large region, 'ordering' it into a relatively small one. For example to create a star which finally explodes as supernova spreading again some of this matter. Another example is that in some cosmological scenarios, our universe will finally collapse back to a singularity. This point somehow remains the Big Bang - extremely small singularity which 'started' our spacetime - with relatively small entropy. Disorder has tendency to grow (four-dimensionally), creating entropy gradient - time arrow.

says that past and future are quite similar - suggests that causality should be able to work in both time directions. In theory used to approximate large-scale physics: general theory of relativity, this similarity is even stronger - time direction is chosen locally and its 'arrow' is chosen just by continuity.

The difference between past and future in physics appears in thermodynamics - its second law says that disorder (entropy) grows with time. This law seems to be in contradiction with CPT conservation: if there is such always growing mathematical property, it should also grow after making CPT transformation - nonsense. From the other side, quantum mechanical evolution is made by unitary operator - cannot increase entropy. We see that the problem is to understand why going from microscopic time scale with unitary evolution, CPT conserving physics to macroscopic time there emerge asymmetry of time directions. The second law has to be understood four-dimensionally. To see it imagine that something is well fixed/ordered/constrained in some point of time - contains small amount of entropy (like Big Bang). Now with time it evolves - somehow loses this constraining and grows with disorder - creating two time arrows: into the future in the future half-plane and into the past in the past one. **Second law of thermodynamics**

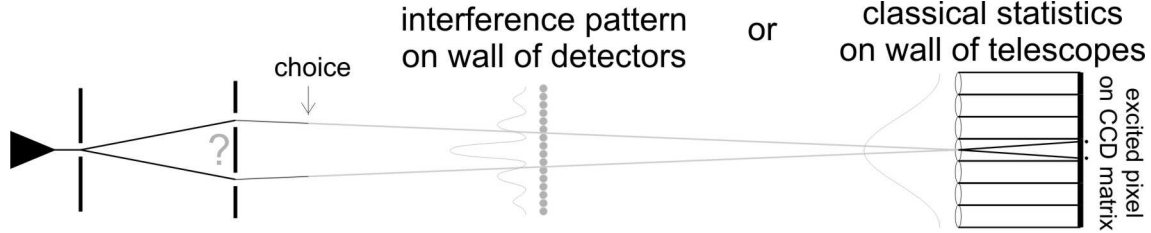


Figure 3: Schematic picture of Wheeler's experiment. If we can determine slit the particle came through, obtained statistics is classical. However, if there is no difference between going through the first and the second one, there appear interference pattern. The idea is that we can determine the slit examining the final trajectory, using for example a telescope. It allows to choose how particle should qualitatively behave while going through double-slit, after it actually did it.

**doesn't have to be directly written in equations ruling our physics, this law is rather a statistical result of their solution we are living in** - with "Big Bang" which created relatively well ordered world - with low entropy (fig. 2).

Let's return to quantum mechanics. We usually use so called Copenhagen interpretation which has been developed to practically work with QM and make us used to inevitability of indeterminism of physics. It also usually uses our natural intuition of time - that against CPT conservation, causality can go only forward in time. However, there is for example Wheeler's experiment ([3]) confirmed recently experimentally ([4]), which shows that we can choose if a photon should behave classically or create interference pattern after it already came through double-slit (fig. 3). This experiment is one of reasons that among a few dozens of developed QM interpretations, some of them treat CPT conservation and so retrocausability seriously. For example in Bohm interpretation ([5]) there are so called pilot-waves (introduced by Louis de Broeglie) which 'goes into the future' to choose current behavior. Transactional interpretation ([6]) takes this symmetry even further - it's formulated using waves going in both time directions.

In this paper we will treat particles not as 'moving points' like in intuitive 'evolving 3D' picture of physics, but as their trajectories in the four-dimensional spacetime like in action optimizing formulation of Lagrangian mechanics. We will see that extremely simplified model - in discredited space with no potential we will get some similarities to quantum mechanics (MERW [7]). While adding potential and taking infinitesimal limit to get continuous space ([8]), we will get a model in which for example electron moving in proton's potential would make some concrete trajectory which average exactly to probability distribution of the quantum mechanical ground state.

These models will suggest further generalizations toward the real physics - that properly constructed classical field theory should be already enough. We will focus on simple generalization of quantum mechanical phase - use classical field of ellipsoids which prefer some shape, but sometimes are enforced to deform because of topological reasons - in the center of particle, giving them some minimal energy: the mass. Thanks of that ellip-

soid axes are not directed, fundamental topological excitations will be three generations of fermions (the number of spatial dimensions), which can continuously transform one into another (neutrino oscillation). We will see that the following expected topological excitations will naturally construct leptons, hadrons and nucleuses with expected mass gradation and decay modes. Later we will quantitatively see that the simplest energy density automatically leads to that rotational degrees of freedom creates Maxwell's equations and gravitation. Noise of ellipsoid shape degrees of freedom should interact extremely weak - can be the source of dark energy. This model is completely symmetric - all asymmetries are results of solution, even the local time direction as in the general relativity theory.

## 2 Maximal entropy random walk (MERW)

We would like to treat particles four-dimensionally: not as a 'moving point', but as its trajectory in 4D spacetime like in Lagrangian formulation. To understand what does it mean we will use some simplified stochastic models. In this section we will focus on the simplest one - using discrete space, time and without potential. We will see that it already leads to intuition from fig. 1. Later we will generalize this model, getting closer to behavior known from quantum mechanics.

In this section we will use discrete space and time. It could be a lattice, but for generality in this moment we will assume that it's just a finite set. Geometry on this space (a graph) is given by a symmetric adjacency matrix. For simplicity we can assume that this graph is connected.

space :  $M_{ij} = 1$  if  $i$  and  $j$  are adjanced, 0 otherwise

time : integer numbers ( $\mathbb{Z}$ )

We can imagine this model that we have some finite set of quantum states, like a lattice of potential wells. Now  $M$  describes possible jumps between them in one time step and we are interested in the whole histories of particle behavior (paths  $\{\gamma_i\}_{i \in \mathbb{Z}}$ ).

Stochastic process usually considered in such spacetime uses assumption that for each vertex, every possible transition is equally probable (Generic Random Walk):

$$S_{ij}^{GRW} = M_{ij}/k_i \quad - \text{probability that after being in } i \text{ we will go to } j$$

where  $k_i = \sum_j M_{ij}$ .

There are possible many different random walks on given graph - given by matrix  $S$  fulfilling  $0 \leq S_{ij} \leq M_{ij}$ ,  $\sum_j S_{ij} = 1$ . Such Markov's processes lead to some stationary probability distribution - dominant eigenvector of  $S$  matrix:

$$\sum_i p_i S_{ij} = p_j \qquad p_i^{GRW} = k_i / \sum_j k_j$$

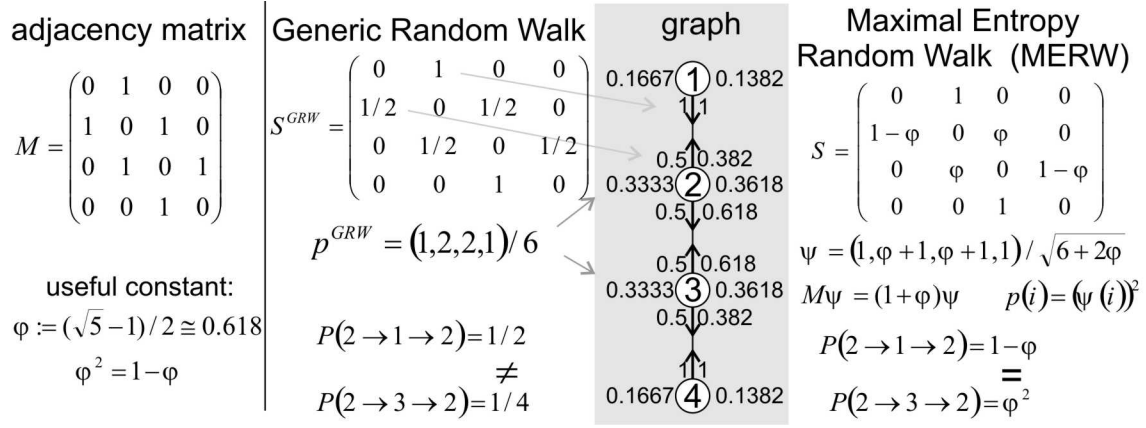


Figure 4: Comparison of generic random walk and maximal entropy random walk for 4 point segment-like graph.

Generic Random Walk seems to be natural, but closer look shows that it emphasizes some paths. For example look at fig. 4: take two infinite paths differing only in one place - one uses  $2 \rightarrow 1 \rightarrow 2$  and the second has in this place  $2 \rightarrow 3 \rightarrow 2$  instead - it makes it twice less probable.

If fundamental objects are paths, there is no reason to emphasize some of them. So the best estimation of their probability distribution should be maximizing entropy uniform distribution (MERW) - **all paths should be equiprobable**. In [8] it is shown that this assumption leads to

$$S_{ij} = \frac{M_{ij} \psi_j}{\lambda \psi_i} \quad p_i = \psi_i^2 \quad \left( \sum_i \psi_i^2 = 1 \right) \quad (1)$$

where  $\lambda$  is the dominant eigenvalue corresponding to  $\psi$  eigenvector ( $M\psi = \lambda\psi$ ). Let's use  $\sum_i \psi_i^2 = 1$  normalization. From Frobenius-Perron theorem we know that this vector is unique and of positive coordinates ( $\psi_i > 0$ ). Derivation of (1) is made by analyzing numbers of finite paths with some fixed part inside and given ending vertices. Now in each step we make iteration step - elongation by one step on both ends and the infinitesimal limit gives us (1). In this paper it is also checked that this random walk really achieve maximal entropy entropy for stochastic processes on given graph.

$$-\sum_i p_i \sum_j S_{ij} \ln S_{ij} \leq \ln(\lambda)$$

Let's look at assumption of uniform distribution among all infinite paths - take two such paths differing only on some finite part. The equality of probability of whole paths gives: **For each two nodes, every path between them of the same length has the same probability**. We can calculate:

$$P(\gamma_0 \gamma_1 \dots \gamma_k) = \frac{1}{\lambda^k} \frac{\psi_{\gamma_k}}{\psi_{\gamma_0}} \quad (S^t)_{ij} = \frac{(M^t)_{ij} \psi_j}{\lambda^t \psi_i}$$

The difference between these equations is  $(M^k)_{ij}$  which counts such equiprobable paths.

To return to quantum mechanics, let's observe that fig. 1 works for MERW with  $S^t$  propagator. Let us assume that some measurement allowed us to fix some vector - in this model it's automatically vector of probabilities of positions. So to estimate probability of a vector in a different measurement, we should just take scalar product - for separate half-planes. Finally the probability is multiplication of both of them. Between both measurements we can eventually make some time evolution, using  $S^t$ .

This time evolution is not unitary like in quantum mechanics, but rather diffusive - increasing entropy up to the stationary distribution, which gives probability distribution as the quantum mechanical ground state. Remember that in macroscopic time, entropy also grows and excited states drop to the ground state - we will discuss it in the next section. Observe that in this model after fixing position in some moment, information about its both later and previous positions decreases with time distance - creates 'time arrows' in both time directions.

### 3 Boltzmann trajectories (BT) and further generalization

In the previous section we've considered a space of all paths in which there was no point of emphasizing any of them, so the best estimation of their probability distribution was maximizing uncertainty uniform distribution. The reality is different - it's more probable that there will be chosen less energetic trajectory. So we should make it able to assign some energy to trajectories. The general choice of behavior is made using potential energy - we will see that using only it, we will get behavior similar to known from quantum mechanics. Kinetic energy terms makes trajectories smoother - without it we intuitively should get some diffusive movement around the expected trajectory. These terms are much more difficult to consider and we will omit them, but theoretically it's possible by creating additional edges with appropriate weights.

Let's fix some potential ( $V_i$ ) in the space. A particle goes through some path means it spends the same time in succeeding potentials - energy of path can be chosen as the sum of succeeding potentials.

Now we can assign energy to trajectories and we want to use it to define statistics among them. In such situation, the natural choice is to use Boltzmann statistics which chooses the best compromise between maximizing entropy and minimizing average energy:

$$\max_{(p_i): \sum_i p_i = 1} \left( - \sum_i p_i \ln(p_i) - \sum_i p_i E_i \right) = \ln \left( \sum_i e^{-E_i} \right) \quad \text{for } p_i \propto e^{-E_i}$$

Originally we can balance between these goals by changing temperature - the larger temperature, the more important entropy is - but it also can be made by rescaling potentials, so we will omit it.

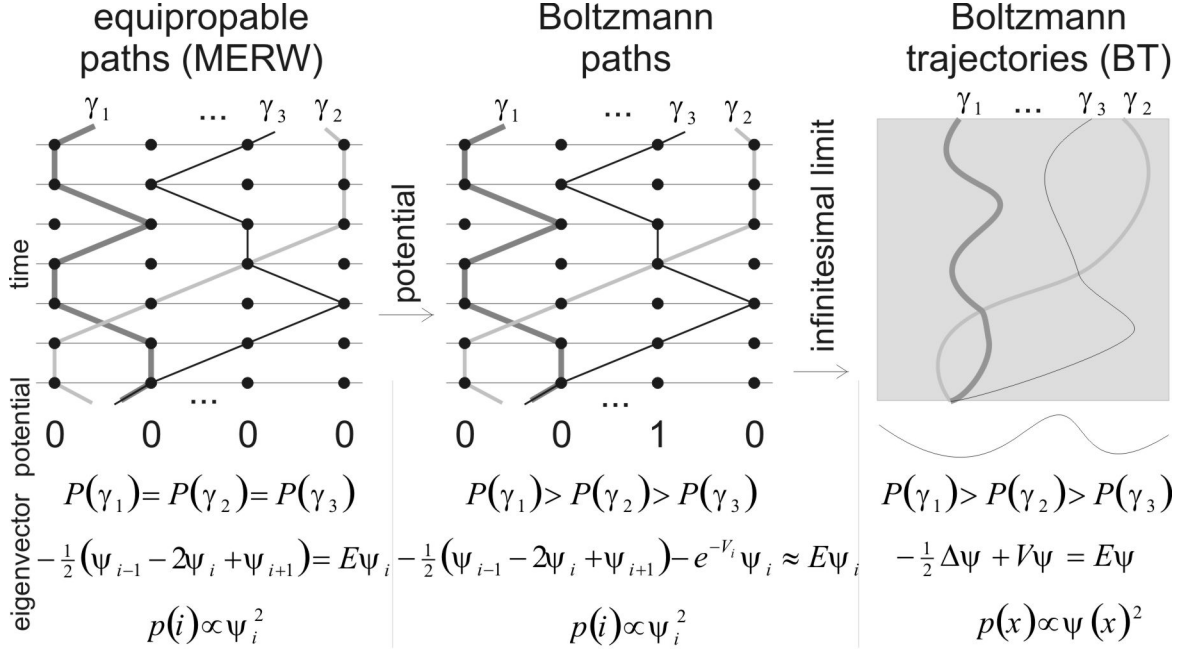


Figure 5: Generalizations of MERW. Introducing potential makes that path staying in energy minimum ( $\gamma_1$ ) is more probable than tunneling path ( $\gamma_2$ ) which is more probable than path staying in energy maximum ( $\gamma_3$ ). Infinitesimal limit is made by decreasing lattice constant down to zero. Eigenvalue equations uses modified eigenvalue  $E = (2 - \lambda)/2$  which allows to write it using discrete laplacian. In this form instead of looking for dominant eigenvalue, we are interested in minimal  $E$  - like for quantum ground state. For Boltzmann paths this equation should really use  $e^{-(V_{i-1}+V_i)/2} + e^{-(V_i+V_{i+1})/2}$  instead of  $e^{-V_i}$ .

To include potential in discrete model, the only difference is to use

$$M'_{ij} = e^{-\frac{1}{2}(V_i+V_j)} M_{ij}$$

instead of the original adjutancy matrix. Now using (1) as previously, we get

$$P(\gamma_0 \gamma_1 \dots \gamma_k) = \frac{e^{-(\frac{1}{2}V_{\gamma_0} + V_{\gamma_1} + V_{\gamma_2} + \dots + \frac{1}{2}V_{\gamma_k})}}{\lambda^k} \frac{\psi_{\gamma_k}}{\psi_{\gamma_0}}$$

While going to continuous case, discrete paths became continuous trajectories (for more details see [8]). We would now intuitively want

$$P(\text{trajectory } \gamma) \propto e^{-\int V(\gamma(t)) dt}$$

To get this limit we can cover the spacetime with discrete lattice and decrease its constant down to zero. However, because of omitting kinetic term, we will get diffusion-like process - to make this infinitesimal limit we have to use characteristic for diffusion processes



time step proportional to the square of lattice constant. We finally get the propagator - probability density of finding particle started in  $x$  in  $y$  after time  $t$ :

$$K(x, y, t) = \frac{\langle x | e^{-t\hat{H}} | y \rangle}{e^{-tE_0}} \frac{\psi_0(y)}{\psi_0(x)} \quad p(x) = |\psi_0(x)|^2 \quad (2)$$

Where  $\psi_0$  is the eigenfunction corresponding to the smallest eigenvalue of Hamiltonian

$$\hat{H}\psi_i = -\frac{1}{2}\Delta\psi_i + V\psi_i = E_i\psi_i \quad \int |\psi_i(x)|^2 dx = 1, \quad E_0 \leq E_1 \leq \dots \quad (3)$$

where  $\Delta = \sum_i \partial_{ii}$ . Formally in this moment we are still working only on real functions and the ground state for each lattice approximation of the spacetime has to be positive, so in infinitesimal limit should be usually also positive (nonnegative).

Returning to quantum mechanics, in this moment we've got that in fixed potential single particle choosing its trajectory according to Boltzmann distribution among all paths will be moving on some concrete trajectory which average to probability density of quantum mechanical ground state. Because of lack of kinetic term, in this model it will be diffusion like trajectory, but generally when we look for example at probability density of electron orbitals, they look to be able to be created by averaging stochastically modified trajectories.

Let's write the propagator in the proper base

$$K(x, y, t) = \frac{\sum_i e^{-tE_i} \langle x | \psi_i \rangle \langle \psi_i | y \rangle}{e^{-tE_0}} \frac{\psi_0(y)}{\psi_0(x)} \quad (4)$$

In this model propagator is not unitary, but increasing entropy up to stationary state. Does it make it nonphysical? Look at excited electron - this state isn't really stable as in quantum mechanics - it wants to deexcitate as in Boltzmann trajectories. We see that second law of thermodynamics suggest that the truth is somewhere in the middle.

The first natural generalization of BT is as it was already suggested by formulas - allowing for complex eigenfunctions. Now 'pure' orbitals are stable and if we add Pauli exclusion principle, according to (4), such many electrons should take up orbits which average exactly to succeeding quantum mechanical orbitals probability density and return back to this configuration after an excitement.

The main advantage of quantum mechanics over BT is the interference, which requires some internal phase of particles rotating while passing spacetime. For internal rotation particle should have rather smooth not diffusive trajectory - the reason of this lack in BT can be that we've omitted the kinetic terms. So the next generalization of BT should be artificially adding imaginary terms to the the exponent corresponding to internal rotations to allow for interference type statistical behaviors.

To go further toward physics as in fig. 6, we see that we need a different approach. There is required a theory in which there naturally appears many long in time and localized

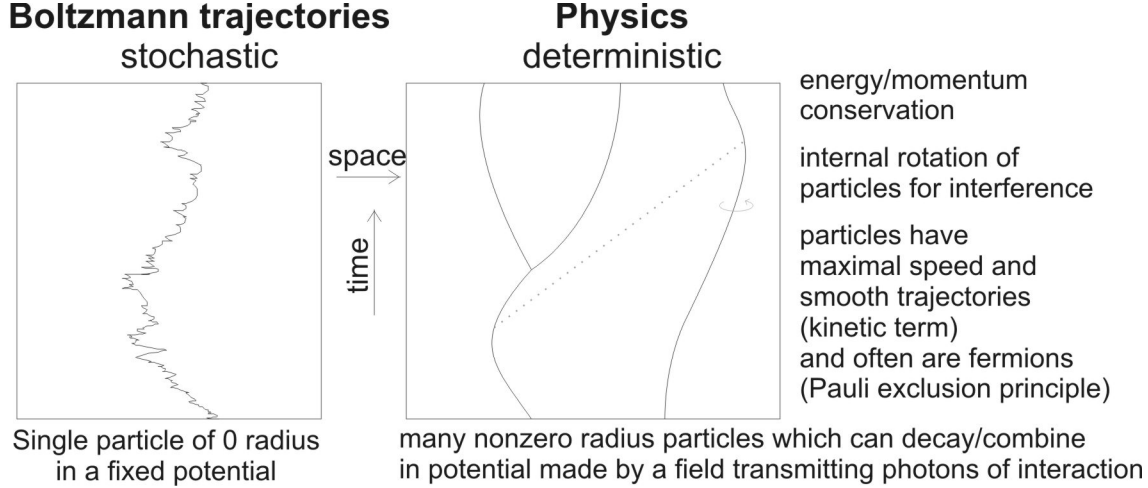


Figure 6: Schematic picture of lacks in Boltzmann trajectories model.

in space phenomenas as particles, conserving momentum/energy, deterministic - that Boltzmann distribution of paths is made due to potentials of other particles. We will see in the following sections that these requirements can be fulfilled by a classical field theory with topological singularities as particles.

In standard approach, managing with multiple particles is made by very successful quantum field theories (QFT). They introduce universal formalism of working with completely abstract particles through creation/annihilation operators and interaction between them. The idea is to give them some concrete form, for example that creation and later annihilation operator creates concrete particle-like local solution between these points in spacetime in the classical field. Analyzing these solutions should give correspondence to QFT, explaining its inconveniences like necessity of cutoffs, renormalizations and let us find new relations between its huge amount of constants. Feynman diagrams from perturbative QFT should correspond to some real scenarios with some probability distribution.

Let's summarize how to combine quantum mechanics with deterministic physics. In presented point of view, particles are not their probability densities, but really goes through some concrete trajectories, which average to these probability densities. Our quantum mechanical description is the result of imperfections of our measurement possibilities, but in fact there are some 'hidden values' which determines further behavior. Observe that there are exceptions from standard quantum measurement - for example we know that in some processes there is created a pair of particles with opposite spin. It made us to add to quantum mechanics concept of entanglement - that there are a few quantum states possible, but we know only probability distribution among them and relations between these probabilities uses squares characteristic for four-dimensional theories.

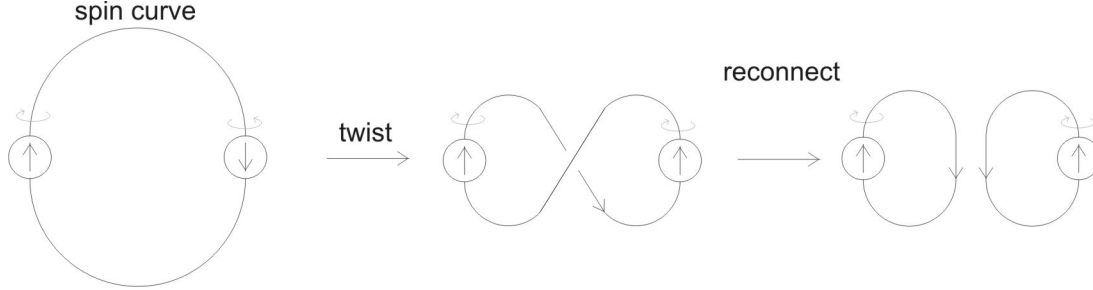


Figure 7: Schematic picture opposite of coupling of opposite spin fermions and some natural way to break such coupling. It requires turning one spin into the opposite one, changing overall spin by 1.

## 4 Topological excitations of ellipsoid field

In this section there will be qualitatively shown preliminary results that model created on taking quantum phase seriously, naturally gives structure of particles and their behavior observed in physics. In the next section will be shown that it also leads to electrodynamics and gravity.

While for example solving Schrodinger's equation for hydrogen atom, there appears characteristic term  $e^{im\varphi}$ , where  $m$  is projection of the orbital angular momentum on the  $z$ -axis and  $\varphi$  is the angle of projection into  $xy$  surface. While making a loop around  $z$  axis, the phase makes  $m$  full rotations. While taking smaller and smaller loops, the phase still makes this rotation - we see that there is a problem on the axis - topological properties makes that no phase can be assigned to it. In mathematics such situation is called topological singularity and this number of rotation is conserved - while deforming the loop, this number is changed only when we cross some singularity. We have such situation e.g. in argument principle form complex analysis or more general Conley index.

Returning to the hydrogen atom, let's call this line around which phase makes rotation **spin curve** - in this case it's  $z$  axis. Generally it can be imagined as 'the middle of tornado' - the curve around which everything rotates. In the solution for atom these curves go into both up and down infinities, but we usually say that quantum amplitude decreases exponentially, so we can forget about these spin curves. I'll try to convince that we cannot base the model on such assumption. Let's imagine that we enclose two such atoms - finally they will have to connect their spin curves, but it's a continuous process - they had to synchronize them and so their phases earlier. Another argument is the way how quantum mechanics is treated with electromagnetism: the fundamental formula is that phase change is  $\int A \cdot dx$  through some path - it strongly suggests that phase is very physical and defined practically everywhere. While using this formula along a loop, we see that because it has to make some integer number of rotations, the magnetic flux is quantified - calculates the number of spin curves going through the loop - we see the conservation law again.

Let's observe that taking phase seriously - that it should be defined everywhere but

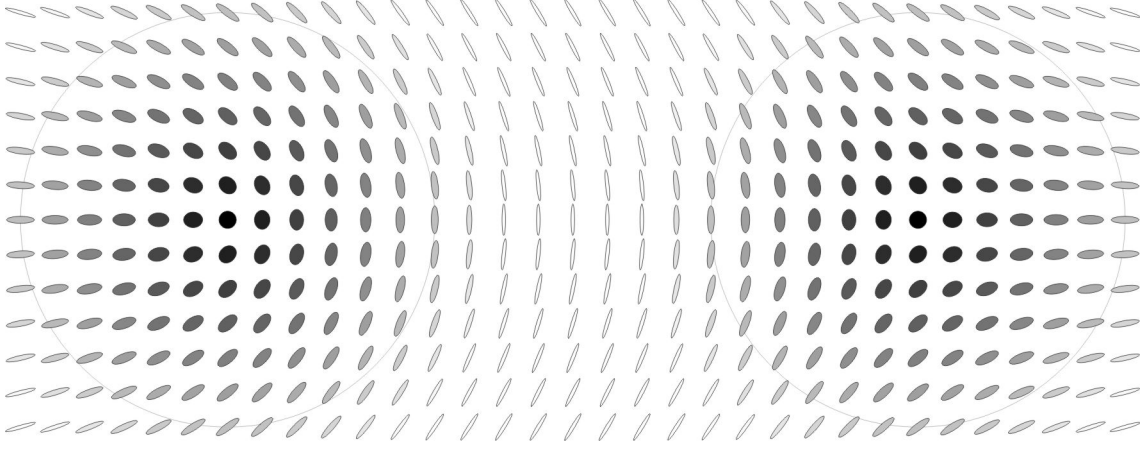


Figure 8: Schematic picture of two simplest topological excitations of ellipse field - fermion-like of opposite  $1/2$  spin - while rotating along grey circles, ellipses make half rotation. In the center of singularity there cannot be emphasized any direction, so ellipses have to deform in a continuous way into a circle. This necessary deformation takes potential out of minimum, giving singularity some minimal energy - the mass. This energy can be released by annihilation - joining with the antiparticle.

the centers of singularities and that spin curves cannot just vanish not meeting with the opposite spin, we will see that coupling of fermions of opposite spins is a natural phenomenon (fig. 7). We can meet them for example on atomic orbitals, in nuclei or as Cooper pairs in superconductors. Two opposite spin curves should attract and finally can reconnect in a different way - the structure of these quanta of magnetic flux should be quite dynamical and complicated. In low temperature such electron couples should be stable - doesn't have to make reconnection of its spin curves with medium's ones, what should transfer part of momentum and causing resistance. On fig. 7 it is also shown a simple way of destroying such couple - by twisting one fermion and reconnecting - now these spin loops can connect with something else or escape far away. This mechanism could be an explanation of that in such processes overall spin often has to be changed by one. For example an excited electron would have to break its coupling to deexcite, changing spin to the opposite one and after all create new couple with corresponding lower electron, explaining selection rules.

Rotation operator in quantum mechanics changes phase, so that while performing the whole rotation the phase makes corresponding number of phase rotations. Look at spin  $1/2$  fermions in this picture - the phase has to become the opposite one. So to work with fermions we should identify phases with the opposite ones. If we would like to represent phase as a vector field, we should forget about their 'arrow' - use direction field instead. Such directions should be defined in practically whole spacetime but the centers of singularities where they cannot emphasize any direction. The simplest topological

excitation of such field are just spin 1/2 fermions. There is simple demonstration showing behavior of phase around a pair of such singularities on a plane [9]. Manipulating their distance we can see that to minimize rotational stress, opposite/the same spins should get closer/farther. In the next section we will see this attraction/repellence as a result of Maxwell's equations.

The problem with vanishing direction in the center of singularity can be solved using ellipse field instead - in each point there is ellipse which prefer to have given radiuses (shape), but sometimes it can deform continually up to circles what cost energy as in fig. 8. For given topological singularity such deformation requires some minimal amount of energy, giving them masses. Looking at spin curves we see that all three directions are essential - it suggest to use ellipsoid field: in each point there are emphasized three directions and they can somehow be deformed to make some of them indistinguishable, but it cost energy. Mathematically it can be represent by using real, symmetric matrix in each point - ellipsoid axes are their eigenvectors and eigenvalues will be corresponding radiuses. Intuitively natural energy density for this model should be

$$\mathcal{E} = \frac{1}{2} \text{Tr} \left( \sum_a \partial_a M \cdot \partial_a M^T \right) + V(M) = \frac{1}{2} \sum_{abc} (\partial_a M_{bc})^2 + V(M) \quad (5)$$

where  $M$  is symmetric, real matrix in each point of spacetime - to shorter notation we will omit argument -  $M$  will always denote  $M(x)$ . The potential term will prefer some fixed shape of ellipsoid, but sometimes will allow for its deformation at cost of potential energy. For example

$$V(M) = \sum_a (\lambda_a - \lambda_a^0)^2 \quad (6)$$

where  $\lambda_a$  are eigenvalues of  $M$  and  $\lambda_a^0$  are some constants of the model and should be all different. In this section we will use three-dimensional ellipsoids, but in the next section we will see that adding fourth dimension will add gravity - this 'time eigenvector' will point local time dimension as in the general relativity theory. This eigenvector is emphasized probably by corresponding to the largest eigenvalue ( $\lambda_0^0$ ), what makes it having the largest tendency to align in one direction. In gravitationally flat spacetime we can omit this forth dimension.

Let's return to the spin curves - observe that they can be made in three canonical ways depending on which eigenvectors makes the rotation. It will result that we will have three generations of particles as in physics. Spin curves are kind of spin 1/2 particles and when they move extremely fast they should be able to go through different ones practically without interaction - like neutrinos. They came in three different versions and by rotation can go one into another, what would explain neutrino oscillation. If we assume  $\lambda_1^0 > \lambda_2^0 > \lambda_3^0$ , it's clear that tau spin curve makes  $\lambda_1 = \lambda_3$ , but for electron and muon spin curves both scenarios are in this moment possible.

Before going to leptons, let's think about the charge. It's also singularity, but different than spins, which were in fact made in two-dimensions orthogonal to the spin curve and in

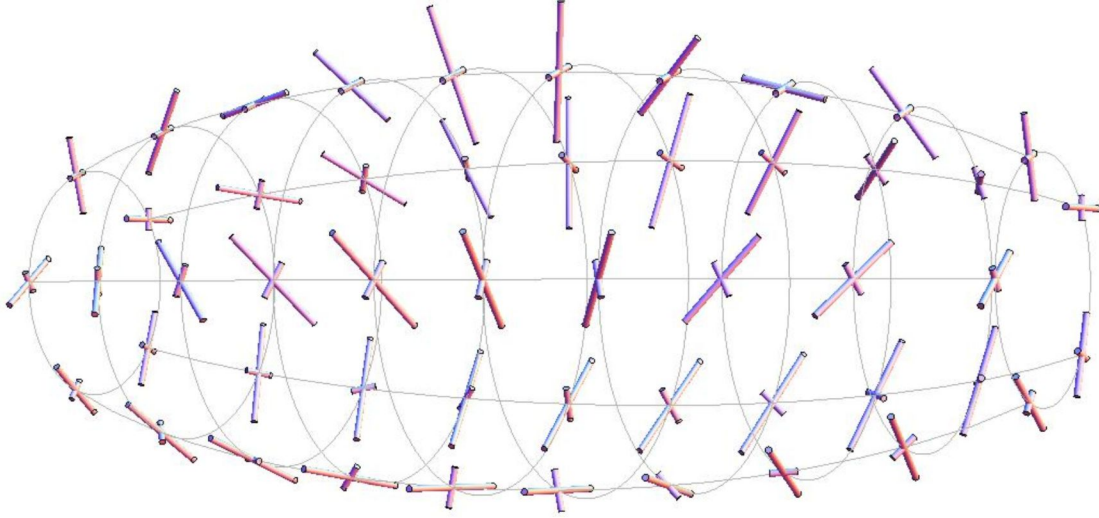


Figure 9: Lepton-like topological singularity: spin curve which makes additional whole rotation towards its center creating the charge. It's antiparticle would make this rotation outward.

three dimensional space has to be 'curve-like'. Charge-like singularities can be 'point-like' and are also conserved - this time the charge surrounded by some area cannot be changed until it passes some singularities - Gauss law shows how to calculate it by integrating over the area. Example of such singularity can be electromagnetic field going in all directions out of particle. In ellipsoid field the following topological excitations are as in fig. 9 - we have outgoing spin curves on both sides and it makes additional full rotation toward the center. Intuitively this singularity creates some additional curvature, giving particle the charge, but it requires further research. This theory allows monopoles - which makes only half rotation, but they should quickly pair with another like creating lepton. This time inside the singularity all three eigenvalues will have to equalize to make eigenvectors indistinguishable - it's much large eigenvalue deformation than for neutrinos, giving them much larger masses. The choice of axes chooses between electron, muon and tau. This model also allows for more complicated topological excitations. Imagine spin curve which encloses to create loop. Normally it should collapse - annihilate releasing stored energy in nontopological excitations (photons). But imagine that before enclosing it makes additional half internal rotation like Mobius strip. This topological restriction forbids it from collapsing and creates much larger and more complicated eigenvalue deformation in the center of this torus-like shape - should be heavier than lepton-like excitations. To decay it can gather charge in one point and repel part of the loop far away - creating lepton and neutrino of the same type. They also don't have spin. All of it makes them very similar to the simplest meson - pion. They usually decays into muons, so such Mobius strip is energetically most preferable for muon-like spin curves. To create kaons it could

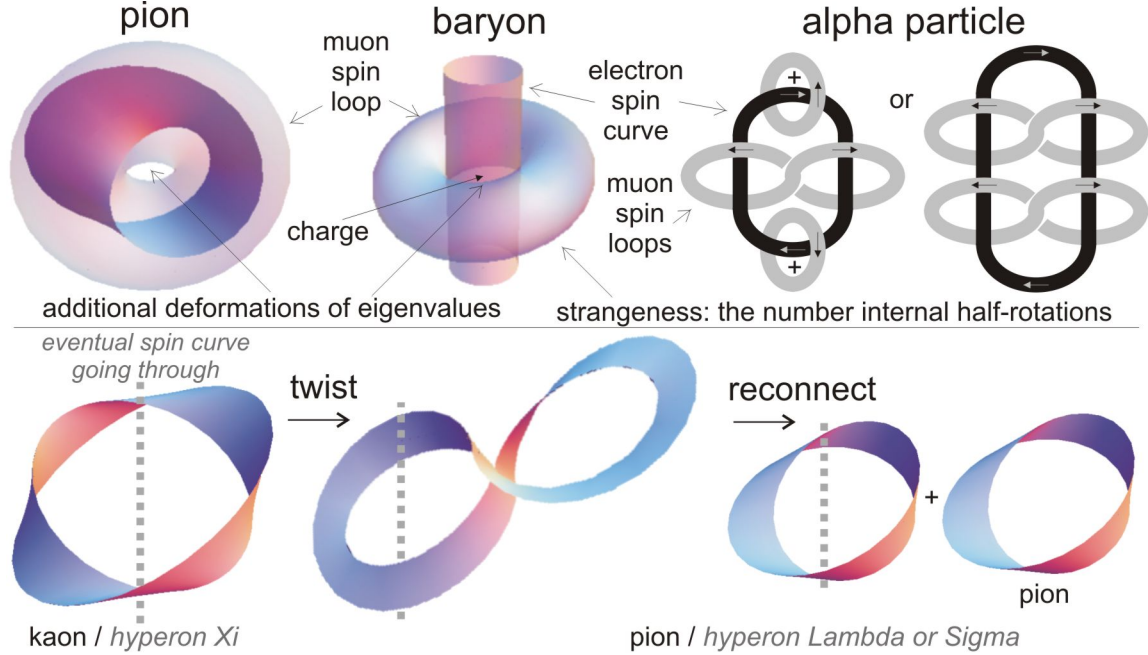


Figure 10: Schematic picture of succeeding topological excitations which are similar to baryons. Pions would be muon spin loop with additional internal half rotation like Mobius strip. Baryons would be electron spin curve going through muon spin loop. Baryons can connect separately their loop and curve parts creating nucleus-like structures. On the lower picture is a general picture of decay processes for loops with internal rotation (strangeness) - internal stress make them twist and then reconnect to release part of it. Other decay processes can be made by creation of charge-anticharge pair on the electron spin curve (beta decay) or expanding the loop (pion decays into muon and neutrino).

make full internal rotation instead of half. Internal stress made by this rotation gives it tendency to twist to 'eight-like shape' or more complicated and reconnect to release some of this internal stress into a pion-like excitations as in physics.

The next possible topological singularity is spin loop around spin curve which forbids it to collapse. This time additional eigenvalue deformation is around the curve - should be heavier than before. Let's find similarities to baryons. On fig. 10 there is schematic picture of hyperon decay - in this way we can explain most of decay modes. Observe that spin loop and spin curve should be made with orthogonal eigenvectors - should be of different type. The decay process suggests that the loop is of muon type, so the spin curve should be lower energetic electron type. Beta decay process suggest that the charge is eventually made in the the spin curve. Such charge seems to be energetically preferable for better placement of some spin loops. The main inconsistency of this model with quark model is the spin of Omega baryon, but predicted  $3/2$  spin hasn't been confirmed experimentally yet. Looking at fig. 10 we see that it could be experimentally interpreted as  $2+1$  structure.

The largest structures we are looking for are nucleus-like. Observe that two opposite spin baryon-like structures should have tendency to attract - now they can reconnect their spin loops, creating 'eight-like shapes' connecting both of them (or more). Their spin curves can also connect, making that there is a few loops around one curve and finally they can construct large and complicated structures, held together by interlacing spin loops and connected spin curves.

## 5 Electrodynamics and gravity

*Notation:*  $i, j, k$  will denote spatial coordinates ( $\sum_i \equiv \sum_{i=1}^3$ ),  $a, b, c$  will denote all coordinates ( $\sum_a \equiv \sum_{a=0}^3$ ). For clarity there will be used no summation convention. Sometime time derivative will be denoted by a dot above (like  $\dot{M}$ ).

In this section we will analyze ellipsoid field quantitatively. There was previously proposed formula for energy density - it's not lagrangian - for example in electrodynamics lagrangian uses  $(E \cdot E - B \cdot B)$ , while energy density uses  $(E \cdot E + B \cdot B)$ . To understand this difference better, let's take a look at the simplest real scalar field:

$$\mathcal{L}' = \frac{1}{2} \left( \partial_0 \phi \partial_0 \phi - \sum_i \partial_i \phi \partial_i \phi \right) - V(\phi)$$

We can define canonical momentums and energy density for it:

$$\pi' = \frac{\partial \mathcal{L}'}{\partial(\partial_0 \phi)} = \partial_0 \phi \quad \mathcal{E}' = \pi \partial_0 \phi - \mathcal{L}' = \frac{1}{2} \left( \sum_a \partial_a \phi \partial_a \phi + V(\phi) \right)$$

such that the energy is time invariant:  $\frac{d}{dt} \int \mathcal{E}' d^3x = 0$ .

Interesting fact is that energy density doesn't emphasize any direction - the choice of time dimension is completely arbitrary. For the selected direction we can find evolution equation using Euler-Lagrange equations such that energy doesn't change while evolution toward that direction.

We can now return to the ellipsoid field. This time energy density should fulfil

$$\mathcal{E} = \sum_{ab} \frac{\partial \mathcal{L}}{\partial(\partial_0 M_{ab})} \partial_0 M_{ab} - \mathcal{L}$$

so to get the expected energy density

$$\mathcal{E} = \frac{1}{2} \text{Tr} \left( \sum_a \partial_a M \cdot \partial_a M^T \right) + V(M) = \frac{1}{2} \sum_{abc} (\partial_a M_{bc})^2 + V(M) \quad (7)$$

we should use

$$\mathcal{L} = \frac{1}{2} \left( \sum_{ab} (\partial_0 M_{ab})^2 - \sum_{abi} (\partial_i M_{ab})^2 \right) - V(M) = \frac{1}{2} \sum_{abcc'} (\partial_c M_{ab}) \eta_{cc'} (\partial_{c'} M_{ab}) - V(M) \quad (8)$$



where  $\eta = \text{diag}(1, -1, -1, -1)$  characteristic for Lorenz invariant theories. Intuitively the first term corresponds to electric field, the second to magnetic field. The potential term is practically nonzero only very near singularities - we will focus on vacuum solutions ( $V = 0$ ) in this moment and use  $M$  in diagonal form

$$M = ODO^T \quad \text{where } D = \begin{pmatrix} \lambda_0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_3 \end{pmatrix}, \quad OO^T = O^T O = \mathbf{1} \quad (9)$$

In fact because of that axes has no "arrows", orthogonal matrix  $O$  can be chosen in  $2^4$  ways, but all of them will give the same results. Differentiating  $O^T O = 1$ , we get useful relation

$$(\partial O)^T O = -O^T \partial O \quad (10)$$

They are asymmetric matrices which defines rotation in given moment - its  $ab$  coordinate describes rotation in  $ab$  plane ( $O \rightarrow O + \epsilon \dot{O} = O(\mathbf{1} + \epsilon O^T \dot{O})$ ). We will use these coordinates as electromagnetic and gravitational fields we are looking for. Let's denote them

$$O^T \dot{O} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -g_1/(\lambda_1 - \lambda_0) & -g_2/(\lambda_2 - \lambda_0) & -g_3/(\lambda_3 - \lambda_0) \\ g_1/(\lambda_1 - \lambda_0) & 0 & -E_3/(\lambda_1 - \lambda_0) & E_2/(\lambda_3 - \lambda_1) \\ g_2/(\lambda_2 - \lambda_0) & E_3/(\lambda_1 - \lambda_2) & 0 & -E_1/(\lambda_2 - \lambda_3) \\ g_3/(\lambda_3 - \lambda_0) & -E_2/(\lambda_3 - \lambda_1) & E_1/(\lambda_2 - \lambda_3) & 0 \end{pmatrix} \quad (11)$$

$$O^T \partial_i O = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -h_1^i/(\lambda_1 - \lambda_0) & -h_2^i/(\lambda_2 - \lambda_0) & -h_3^i/(\lambda_3 - \lambda_0) \\ h_1^i/(\lambda_1 - \lambda_0) & 0 & -B_3^i/(\lambda_1 - \lambda_0) & B_2^i/(\lambda_3 - \lambda_1) \\ h_2^i/(\lambda_2 - \lambda_0) & B_3^i/(\lambda_1 - \lambda_2) & 0 & -B_1^i/(\lambda_2 - \lambda_3) \\ h_3^i/(\lambda_3 - \lambda_0) & -B_2^i/(\lambda_3 - \lambda_1) & B_1^i/(\lambda_2 - \lambda_3) & 0 \end{pmatrix}$$

Thanks of this we can simply write derivatives of  $M$  in the chosen base:

$$O^T \dot{M} O = O^T (\dot{O} D O^T + O D \dot{O}^T) O = (O^T \dot{O}) D - D (O^T \dot{O}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & g_1 & g_2 & g_3 \\ g_1 & 0 & E_3 & E_2 \\ g_2 & E_3 & 0 & E_1 \\ g_3 & E_2 & E_1 & 0 \end{pmatrix}$$

Where we've used (10). Frobenius norm doesn't depend on the choice of base, so we get

$$\sum_{ab} (\partial_0 M_{ab})^2 = \text{Tr}(\dot{M} \dot{M}^T) = \text{Tr}(O^T \dot{M} O O^T \dot{M}^T O) = E \cdot E + g \cdot g$$

and analogously for spatial derivatives. Finally the lagrangian in these coordinates is

$$\mathcal{L} = \frac{1}{2} \left( E \cdot E - \sum_i B^i \cdot B^i \right) + \frac{1}{2} \left( g \cdot g - \sum_i h^i \cdot h^i \right) - V(M) \quad (12)$$

where  $E = (E_1, E_2, E_3)$ ,  $g = (g_1, g_2, g_3)$ ,  $B^i = (B_1^i, B_2^i, B_3^i)$ ,  $h^i = (h_1^i, h_2^i, h_3^i)$ .

We get flat spacetime (no gravity) when local time eigenvector is along dimension we are making evolution (zeroth coordinate) - in this case  $g$  and  $h$  vanishes. If we introduce new coordinates

$$(B_j)^2 = (B_j^1)^2 + (B_j^2)^2 + (B_j^3)^2 \quad (13)$$

we will get exactly lagrangian which leads to electromagnetic four potential and finally to Maxwell's equations in vacuum. While making this transformation there is a problem with choosing signs to achieve continuity, but anyway we have guarantee that both lagrangians - (12) and Maxwell's in reduced coordinates gives the same behavior. This forgetting of some internal degrees of freedom allows for not seen in electromagnetism particle's internal rotation required for interference - (11) gives such rotation depending on particle's electromagnetic properties.

In this model gravity also leads to corresponding Maxwell's equations - with mass density instead of charge density and opposite sign in potential equation to make it always attracting. We also get kind of Ampere's law for gravity, which can be seen as a result of Lorenz invariance as in electrodynamics. In this theory electromagnetic and gravitational waves has always constant speed and don't interact in perfect vacuum. On the end of this section will be discussed if it still can lead to general relativity effects, especially that there is no perfect vacuum in reality.

There left to show that the field around particles gives them magnetic momentum and charge. While making a loop around spin curve,  $O$  makes half rotation - it enforces nonzero  $B$  going through this loop. Charged particles are much more complicated, precise calculations will rather require simulations. Intuitively charge/mass inside some volume corresponds to integral of curvature around it, so  $\Delta M_{00}$  should correspond to mass density,  $\Delta M_{ii}$  to charge density. We will see that deformation of eigenvalues enforces nonzero laplacian of diagonal terms of  $M$ .

We will now take a look at situation near singularities, for example to find gravitational mass. Looking at variations for all  $a, b$  we get Euler-Lagrange equations for this case

$$\sum_c \partial_c \frac{\partial \mathcal{L}}{\partial (\partial_c M_{ab})} = \frac{\partial \mathcal{L}}{\partial M_{ab}}$$

The right side depends only on the potential, let us assume  $V(M) = \frac{1}{2} \sum_a (\lambda_a - \lambda_a^0)^2$

$$\frac{\partial \mathcal{L}}{\partial M_{ab}} = \frac{\partial V}{\partial M_{ab}} = \sum_c \frac{\partial V}{\partial \lambda_c} \frac{\partial \lambda_c}{\partial M_{ab}} = \sum_c (\lambda_c - \lambda_c^0) O_{ac} O_{bc} = (O \cdot \text{diag}(\lambda_c - \lambda_c^0) \cdot O^T)_{ab}$$

where  $\frac{\partial \lambda_c}{\partial M_{ab}}$  can be calculated by perturbation of eigenvalue equation. Finally we get

$$\partial_0 \partial_0 M_{ab} = \sum_i \partial_i \partial_i M_{ab} - (O \cdot \text{diag}(\lambda_c - \lambda_c^0) \cdot O^T)_{ab} \quad (14)$$

$$\ddot{M} = \Delta M - O \cdot \text{diag}(\lambda_c - \lambda_c^0) \cdot O^T \quad (15)$$

For simplicity we will look at the middle of singularity in its center of mass coordinates, which were additionally chosen to diagonalize  $M$  ( $O = \mathbf{1}$ ). In the center two or three spatial eigenvalues equalize and we would like it to cause curvature of the time eigenvector field toward the mass. The singularity is not moving, so while evolution time eigenvector shouldn't change:  $\dot{M}_{00} = 0$ :

$$0 = \Delta M_{00} - (\lambda_0 - \lambda_0^0) \quad (16)$$

While going out of the center, we would like time eigenvector to bend toward the center to create gravity:  $\Delta M_{00} < 0$ . That means that  $\lambda_0$  should be smaller than  $\lambda_0^0$ . In this moment there is no point for such behavior - to create gravitational mass we rather have to modify the model to reduce time eigenvalue while deforming spatial ones. It can be made in many ways. Intuition suggest doing it by physically looking assumption, especially that there still could be some lower order theory behind, that the volume of ellipsoids doesn't change:

$$\lambda_0 \lambda_1 \lambda_2 \lambda_3 = \det(M) = \text{const} \quad (17)$$

While deforming spatial eigenvalues toward some average value, their product increases - this restriction makes that  $\lambda_0$  has to decrease. The larger  $\lambda_0^0$ , the smaller this compensation has to be - weakness of gravity force suggest that this eigenvalue is much larger than spatial ones.

There left to think about general relativity effects in this model. They probably could be added by some additional intrinsic curvature as it is usually made, but it suggest that our spacetime is some infinitely thin surface embedded in some noninteracting multidimensional medium and could create nontrivial topology, like wormholes. A surface with constant positive curvature has to enclose into a sphere, but where it's happening? I believe that we should look for alternative to general relativity, which doesn't need intrinsic curvature. It require further research, but presented ellipsoid field could give an answer.

How to create general relativity effects in model with constant electromagnetic waves speed? Time dilation says that in gravitational potential everything happens faster. In usual matter practically all interactions are made by these waves - making everything happen faster can be made by reducing distances - we would need that curvature of time eigenvector field made the whole matter to 'rescale' for example by changing local masses, charges and magnetic momentums. We couldn't observe it directly - because our measurement apparatus is made of the same matter, we would observe only gravitational redshift while connecting instruments from different gravitational potential.

Look at particles in this model with nonzero time eigenvector field curvature. While making loop around spin curve, given spatial eigenvector is twisted a bit due to curvature - only part of it actually makes the rotation - charges and magnetic momentums should be reduced a bit. From the other side eigenvalue deformation have to be made on a bit smaller space - it should increase masses a bit. These local modifications of physical constants could make that electron orbits are a bit narrower, chemical molecules are a

bit smaller and finally the whole matter is in the first approximation rescaled, making all distances a bit smaller.

Another general relativity effect is gravitational lensing. Maybe it could be explained by that the vacuum isn't completely empty - it contains quite a lot electromagnetically interacting particles. There is some interaction of electromagnetic field with this medium, it could transfer part of its acceleration toward the center of mass.

Another difficult to explain in current theories is the cosmological constant required to explain astronomical observations. In ellipsoid model this energy density could be explained by small vibrations of eigenvalues. Standard interactions uses only rotational degrees, so noise in these ellipsoid shape degrees of freedom should interact extremely weak - only while particle creation/annihilation. Universe had billions of years to thermalize these degrees of freedom with observed 2.7K electromagnetic noise. This noise could also help light being attracted gravitationally.

We see that for more precise description and understanding of this model there will be needed numerical simulations.

## References

- [1] J. S. Bell, *On the problem of hidden variables in quantum mechanics*, Rev. Mod. Phys. 38, 447 (1966),
- [2] J. Schwinger, *The Theory of Quantized Fields. II*, Harvard University, United States Department of Energy (through predecessor agency the Atomic Energy Commission), (1951),
- [3] J. A. Wheeler, *The 'Past' and the 'Delayed-Choice Double-Slit Experiment'*, Mathematical Foundations of Quantum Theory, Academic Press (1978)
- [4] V. Jacques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect, J.-F. Roch, *Experimental realization of Wheeler's delayed-choice GedankenExperiment*, <http://arxiv.org/abs/quant-ph/0610241>
- [5] D. Bohm, *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden Variables" I*, Physical Review 85: 166–179,
- [6] J. Cramer, *The Transactional Interpretation of Quantum Mechanics by John Cramer*, Reviews of Modern Physics 58, 647-688, July (1986),
- [7] Z. Burda, J. Duda, J. M. Luck, B. Waclaw, *Localization of the Maximal Entropy Random Walk*, Phys. Rev. Lett. 102, 160602 (2009),
- [8] J. Duda, *Optimal encoding on discrete lattice with translational invariant constraints using statistical algorithms*, <http://arxiv.org/pdf/0710.3861>,
- [9] <http://demonstrations.wolfram.com/SeparationOfTopologicalSingularities/>